Minia Journal of Engineering & Technology (MJET), Vol. 38, No. 1. January 2019

THEORETICAL ANALYSIS OF THE BEHAVIOR OF HIGH STRENGTH REINFORCED CONCRETE BEAMS UNDER FLEXURE

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Abstract

The flexural behaviour of High Strength Concrete shallow beam depends on many factors such as, span –to- depth ratio, shear span –to –depth ratio and the percentage of tension steel. This paper presents theoretical study of flexural behavior high strength reinforced concrete beams using the well-known three dimensional nonlinear finite element program (ANSYS release 11) aiming to examine the effect of these factors on the behaviour of HSC. This study includes thirteen beams. All studied beams were simply supported under two point loads and had the same concrete strength of 750kg/cm^2 and the same thickness of 45 cm. The beams were divided into three groups depending on the studied parameters. The variable in the first group was the span –to –depth ratio, in the second group was the shear span -to–depth ratio and in the third group was the percentage of tension steel.It was found that the increasing of percentage of span to depth ratio has a major significant effect on ductility but has a negligible effect on ultimate moment, increasing of percentage of tension steel has a minor effect on ductility but has a major significant effect on the ultimate moment, the cracking moment was effected by the increasing of percentage of tension steel which is more than the balanced. Also it was found the Increasing of the shear span to depth ratio has a major significant effect on the shape of failure.

Introduction

High strength concrete (HSC) offers many advantages over conventional concrete, so it used effectively in compression members such as columns, piles, domes, folded plates, shells and arches where large in-plane compressive stresses exist. Despite a large number of research activities (5, 6, 8, 11 and 13) that has concerned the flexural behaviour of High Strength Concrete beams controversy still remains with regard to some vital design issues. One such issue is the serviceability requirement of deflection. Beams tested by several investigators consistently demonstrated significantly larger deflections at service load than what would be predicted by following the ACI Code16 provisions. Even the assumption of cracked moment of inertia as the effective value and use of the representative expressions for the elastic modulus of concrete as reported by ACI Committee 36317 for HSC had failed to bring the predictions on the conservative side. Therefore, explanations must be sought through further investigations. Another important design issue is the ductility or the ability of a reinforced concrete (RC) member to deform at or near the ultimate load without significant strength loss. Because concrete becomes increasingly more brittle as its compressive strength is increased, guaranteeing adequate ductility represents one of the primary design concerns when HSC is involved.

1- Immediate deflection of Beams

The immediate deflection in beams is depending on bending moment, shear force, axial force and shrinkage of concrete. In this study the deflection due to shrinkage was not included. The moment curvature relationship is given by the following second order equation:-

$$M = EI \frac{\partial^2 w}{\partial^2 x} \qquad (1)$$

Where EI = flexural rigidity, w= lateral deflection.

Integration of the moment –curvature relationship satisfying the prescribed beam condition gives the value of w. This deflection resulting solely from curvature changes is called the bending deflection.

The sheer force Q causes shear stress τ that is non-uniformly distributed over the cross section. The stress τ has a maximum value at the neutral axis, the distortion strain of the cross section is also not constant and an average value for the whole section is given by

$$\frac{\delta w}{\delta x} = \frac{\tau}{G} = \gamma \frac{Q}{AG} \qquad (2)$$

Where A, is area of cross section, G is the shear modulus, γ a factor to reflect the effect of non-uniform distribution of shear stress on the average distortion. The displacement w, resulting from the distortion caused by shear force is called the deformation due to shear.

The axial force F causes an axial normal stress σ_o and a net axial displacement u_p. In addition to these effects, the axial force causes a bending moment due to the eccentricity of the axial load with respect to the deformed position of structure. This effect makes the load deformation behavior of structure non –linear. The total displacement is sum of displacement is sum of displacement due to change in curvature caused by bending M and the displacement resulting from the distortion caused by shear Q. Generally the bending deformation is the major component of the total displacement except in the case of beams with low span to depth ratio.

2- Nonlinear Finite Element Analysis of the studied beams

The theoretical study of this work was performed by using well known structure computer program call Ansys, based on Finite Element Method. Eight-node three dimensional elements having six degree of freedom at each node: three translations and three rotations in the nodal x, y and z direction. The most important aspect of this element is the treatment of nonlinear material .The element is capable of plastic deformation, cracking in three orthogonal directions and crushing. Link 8 is used in Ansys to model the steel reinforcements. Two nodes are required for this element. Each node has three degree of freedom, translations in the nodal x, y and z direction. The Link 8 element is an uni-axial tension –compression element. As in a pin-jointed structure, no bending is considered .The reinforcement steel elements are connected to concrete elements at nodal points. Composite action is enforced through compatibility at the nodes.

3- Numerical parameters influence the Finite Element Solution

The usefulness of the finite element method for nonlinear analysis very much depends on the various numerical which influence the solution. Such parameters can be classified in to three groups, solution parameters, quasi-material parameters and actual material.

3-1 Solution parameters

Solution parameters are convergence factors and criteria, number of iteration required to achieve in acceptable solution, method rule of updating the stiffness, order of gauss rule and mesh size. The full Newton –Raphson method is the iterative process adopted by Ansys in solving the nonlinear equations. In this method for each trial the stiffness matrix is modified to include the effect of plastic strain increments computed at integration points. Two by two gauss points rule were recommended by the manual user of the program for this considered problem. A large number of these elements were used in this analysis to satisfy the details of steel .In this analysis the convergence tolerance was taken at 0.001 of displacement with maximum iteration number of 25, to reduce the accumulation forces within the iteration.

3-2 Quasi- Material parameters

Quasi- Material parameters are hardening rules which describe the change of position of the subsequent yielding surface during plastic deformation, concrete tension cracking modeling aggregate interlocking effects and tension stiffing effects. Ansys provides a kinematics- hardening rule which assumes that the yield surface remains constant in size but translates in stress space with progressive yielding fracture criterion in concrete. In Ansys, the elastic strain hardening model is combined with a five parameter failure criterion for a complete characterization of the concrete material behavior, and it is given as:

$F/f_c - S \leq 0$ (3)

Where, F is a function of principle stress state, S is failure surface expressed in terms of principle stresses and five parameters, f_t (ultimate uni-axial tensile strength, f_c(ultimate uni –axial compressive strength), f_{cb}(ultimate bi-axial compressive strength), f_1(ultimate compressive strength for a state of bi-axial compression superimposed on hydrostatic stress state), f_2(ultimate compressive strength for a state of uni-axial compression superimposed on hydrostatic stress state). In Ansys program, fracture of concrete can occur in two different ways, cracking type of fracture and crushing type of fracture. The first one occurs if one of the principal stresses is tension and exceeds the limiting value. Smeared cracking model is generally used for most structural application, especially if the overall load deflection behavior is desired, which used in this work. Also in this analysis there is the possibility of a crack opening and closing crack opening. After crack has formed the crack that results is rough, and capable to transferring some shear. In the present work two values are specified for transfer of shear in cracked element, one of the closed crack (0.8) and the other for an open crack (0.2).To improve the convergence criteria, the tension stiffening is considered in this analysis and its factor was taken equal to (0.6).

3-3 Actual Material parameters

Actual material parameters are crushing strength concrete, tensile strength of concrete, young's modulus of concrete and steel reinforcement.

4- Verification of Ansys results

The theoretical research described in this study was certificated in reference [8]

5- Theoretical program

The beams were divided into three groups. The variable in the first group was the span –to –depth ratio, in the second group was the shear span -to–depth ratio and in the third group was the percentage of steel in tension. Table (1) and figure (1) show details of the theoretical models.

Table (1) Details of the numerical models

group	Beam N0	span (m)	width (m)	Total depth (m)	span / Total depth	Stirrups /m'	shear span / Depth	(u)	(u')	α
group (1) main variable (length)	A_{11}	4.0	0.25	0.45	8.88	5Φ8	4.5	2.5	0.33	0.13
	A_{12}	5.0			11.11					
	A_{13}	6.5			14.44					
	A_{14}	7.5			16.66					
	A_{15}	9.5			21.11					
group (2) main variable shear - depth ratio	A_{21}	6.5	0.12		14.44	zero	1.0	2.5	zero	zero
	A_{22}						2.0			
	A_{23}						3.0			
	A_{24}						4.0			
group (3) main variable percentage of steel in tension	A_{31}	6.5	0.12		14.44	5Φ8	4.5	1	0.56	0.56
	A_{32}							2.5		0.22
	A_{33}							4		0.14
	A_{34}							6		0.09

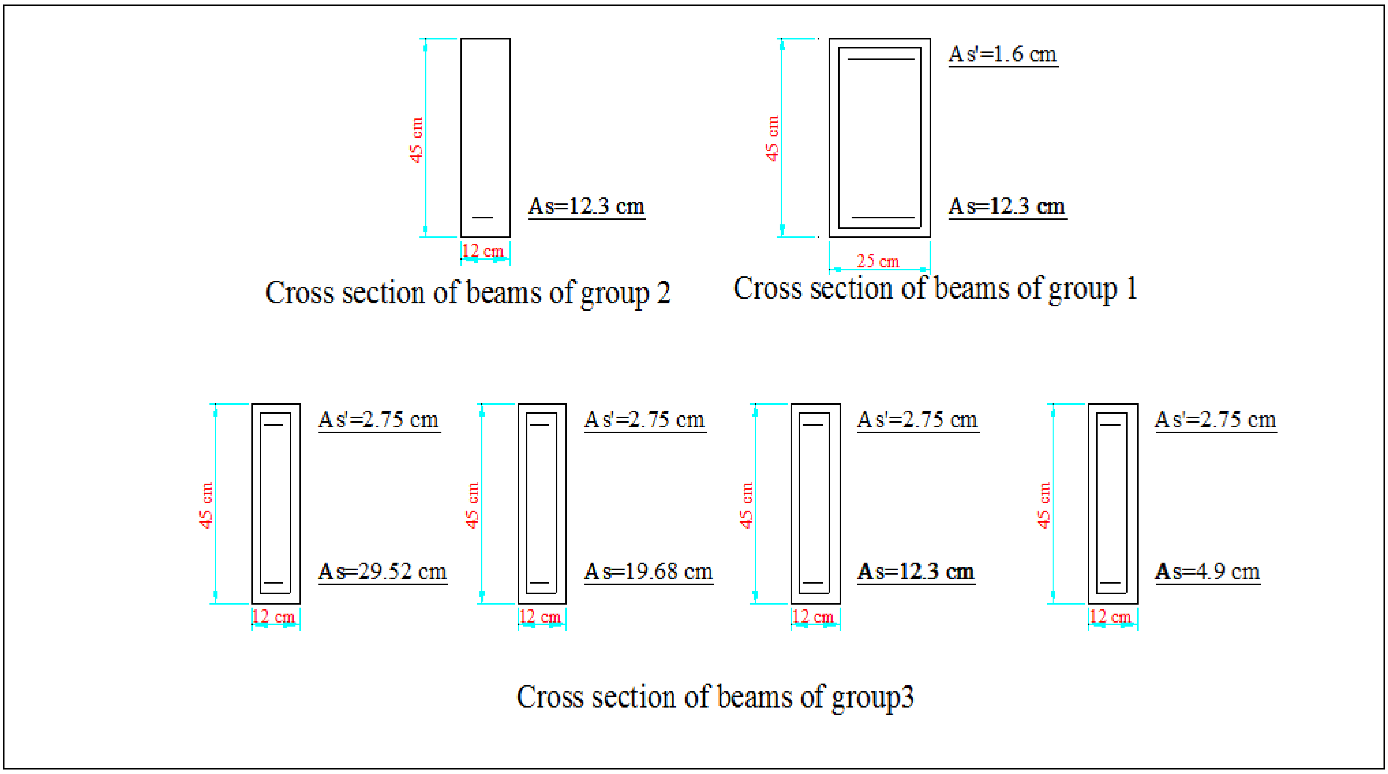


Figure (1) Cross sections of the analyzed beams

5-1 Group one (span – to- total depth ratio):-

a- *Details of the analyzed beams of group one*

This group consists of five beams; they were identical in cross section, steel reinforcement, and shear span –depth ratio as shown in table (1).The length of the beams were 4.0, 5.0, 6.5, 7.5 and 9.5m for beams A_{11}, A_{12}, A_{13}, A_{14} and A_{15}, respectively with corresponding span- to total depth ratio of (8.88,11.11, 14.44,16.66 and 21.11) respectively. The width of the beams =0.25m.Figure (2) presents the geometry and the load of the beams, in addition to the finite element idealization.

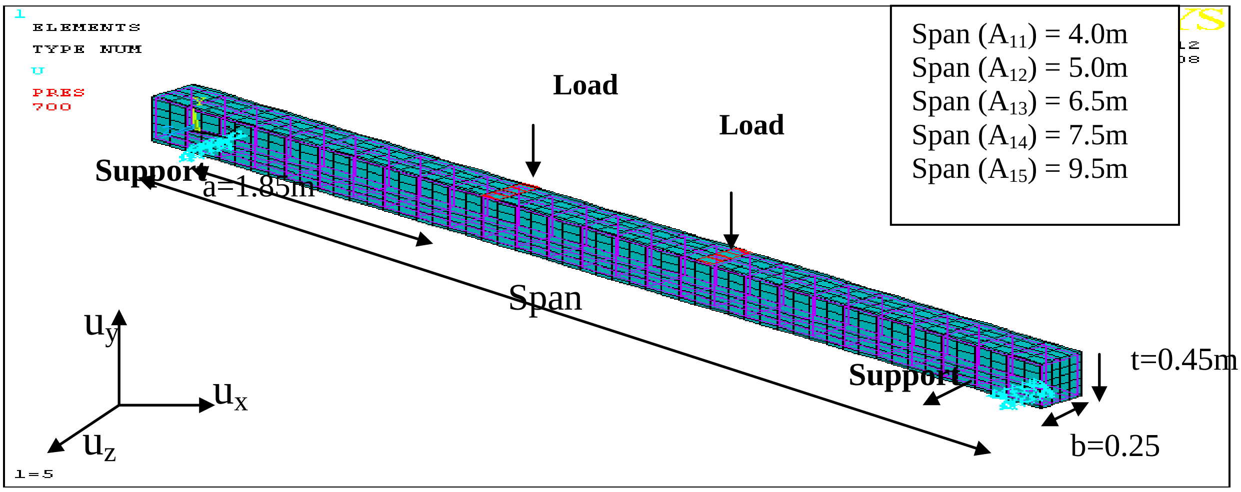


Figure (2) Finite element idealization of beams of group one

b- Cracks pattern

Since all the analyzed beams of this group were identical in cross section, compressive strength, percentage of tension, compression steel and same shear span –to-depth ratio, so it had the same general shape of crack pattern as shown in figure (3). The first crack load appeared at the maximum bending moment zone (at mid span). It can be seen from this figure that the mode of failure of this group was flexural failure, as all cracks initiated at the bottom surface in the mid part of the beam and extended up ward thought the web. At last stages of loading some diagonal cracks appeared at the part between the load and the support.

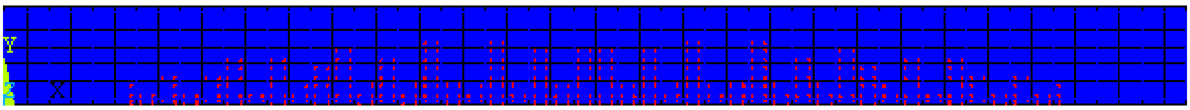


Figure (3) Cracks pattern for beams of group one

c- The moment –deflection relationship

The variations of deflection with moment of all analyzed beams with different values of span –to total -depth ratio (8.88, 11.11, 14.44, 16.66 and 21.11) are shown in figure (4). Table (2) summarizes the theoretical results of the analyzed beams of this group. The effect of beam size can be seen clearly on the response of the analyzed beams to the load, as span –to-depth ratio increases the deflection increases. It can be concluded that the maximum deflection depending on the span –to- total depth ratio as shown in figure (5).

d- Code limitation for deflection

In ECP203 (Egyptian code of reinforced concrete design) for normal concrete, deflection of simple supported beams not need to be calculated if the span less or equal 10 meter and span –to-total depth ratio($\frac{l}{t}$ ≤16) but if the span more than 10 meter deflection should be calculated. Using the same rule of span–to-depth ratio, comparison was made between the predicated deflection at 0.67 of ultimate theoretical load which is considered as service load and the value recommended by code based on (span /250) for μ (Percentage of tension reinforcement) =2.5 % and μ' (Percentage of compression reinforcement)=0.33 as given in table (2). It can be seen from this comparison that the predicted value of deflection at o.67 of ultimate moment capacity less than recommended value based on the (span/250) up to span –to- total depth ratio =14.44. For spans 7.5 m ($\frac{l}{t}$ =16.66) and span 9 m ($\frac{l}{t}$ =21.11), the results obtained from the finite element were more than the recommended by code because of $\frac{l}{t}$ ≥16 not recommended in ECP203.

Table (2) summarizes the theoretical results of the analyzed beams of first group

Beam No	L (m)	$\frac{l}{t}$	(M_{cr}) (m.t)	(M_u) (m.t)	$\frac{Mcr}{Mu}$	Shape of failure	δ_{cr} (mm)	δ_s (mm)	δ_{code} (mm)	$\frac{\delta_s}{\delta_{code}}$
A_{11}	4.0	8.88	8.785	37.48	0.230	flexural failure	1.41	12.26	16	0.76
A_{12}	5.0	11.11		36.10	0.242		2.47	18.15	20	0.9
A_{13}	6.5	14.44		35.64	0.246		4.52	25.19	26	0.96
A_{14}	7.5	16.66		35.64	0.246		6.17	34.9	30	1.16
A_{15}	9.5	21.11		35.64	0.246		10.2	59.28	38	1.55
L=Length, t= total depth, M_{cr}=cracking moment, M_u=ultimate moment, δ_{cr}= deflection at cracking moment, δ_s= deflection at 0.67 of ultimate moment, δ_{code}= deflection recommend by code (L\250)										

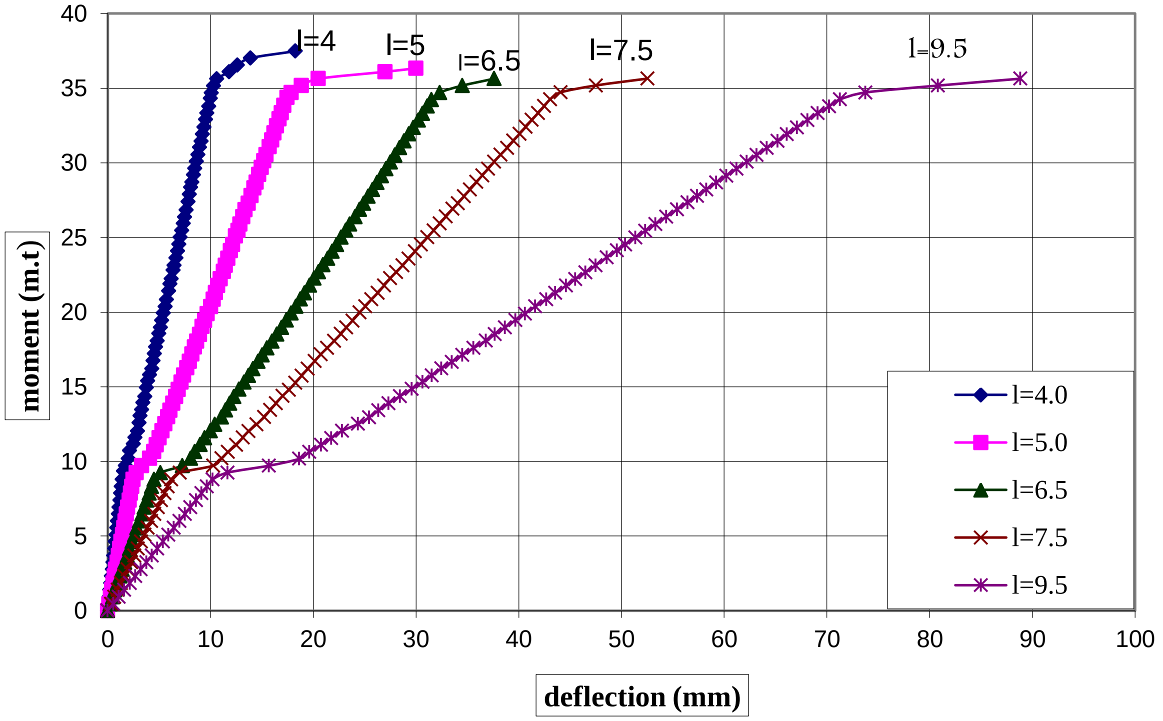


Figure (4) Moment-deflection relationship of beams of first group

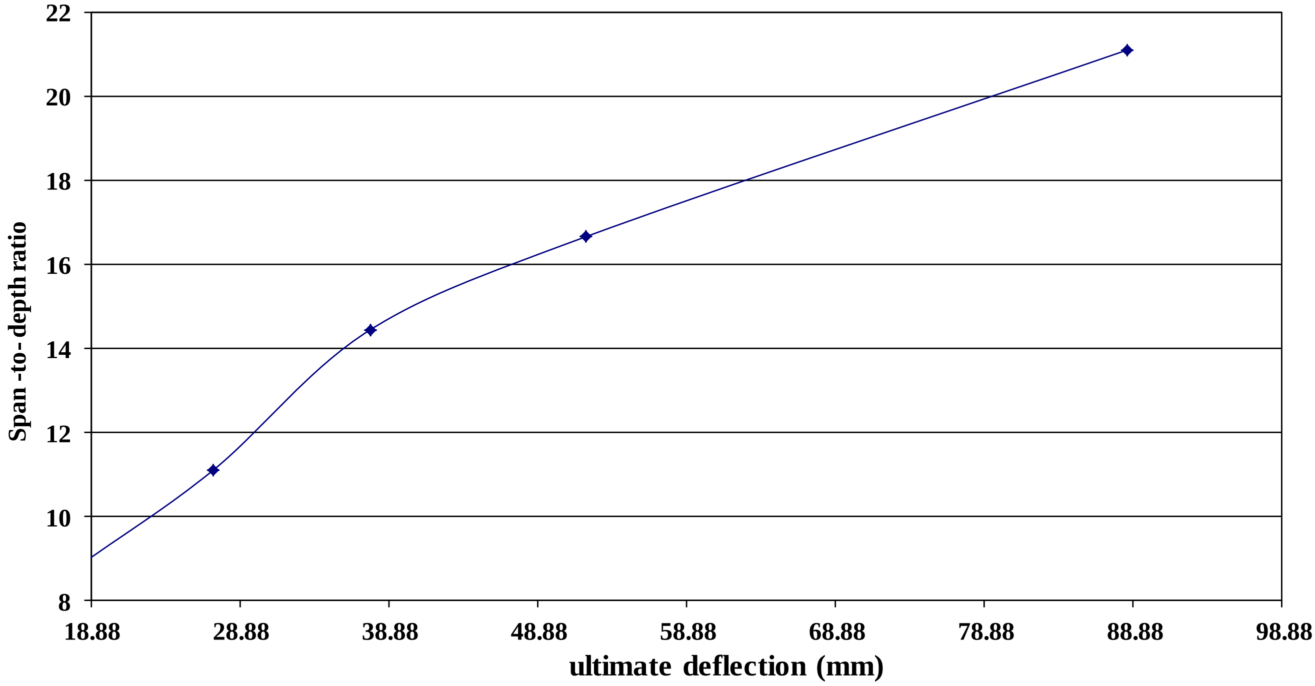


Figure (5) Reletion between ultimate deflection and span-to- total depth ratio

5-2 Group two (shear span- to –depth ratio):-

a- *Details of the analyzed beams of group two*

This group consisted of four beams; they were identical in cross section, steel reinforcement, and the length. The main variable was the shear span- to- depth ratio. Four values of shear span- to- depth ratio were considered 1, 2, 3, and 4 for beams A_{21}, A_{22}, A_{23} and $A_{24,}$ respectively. Table (1) shows the details of the analyzed beams. Figure (6) presents the geometry, the load of the beams and the finite element idealization.

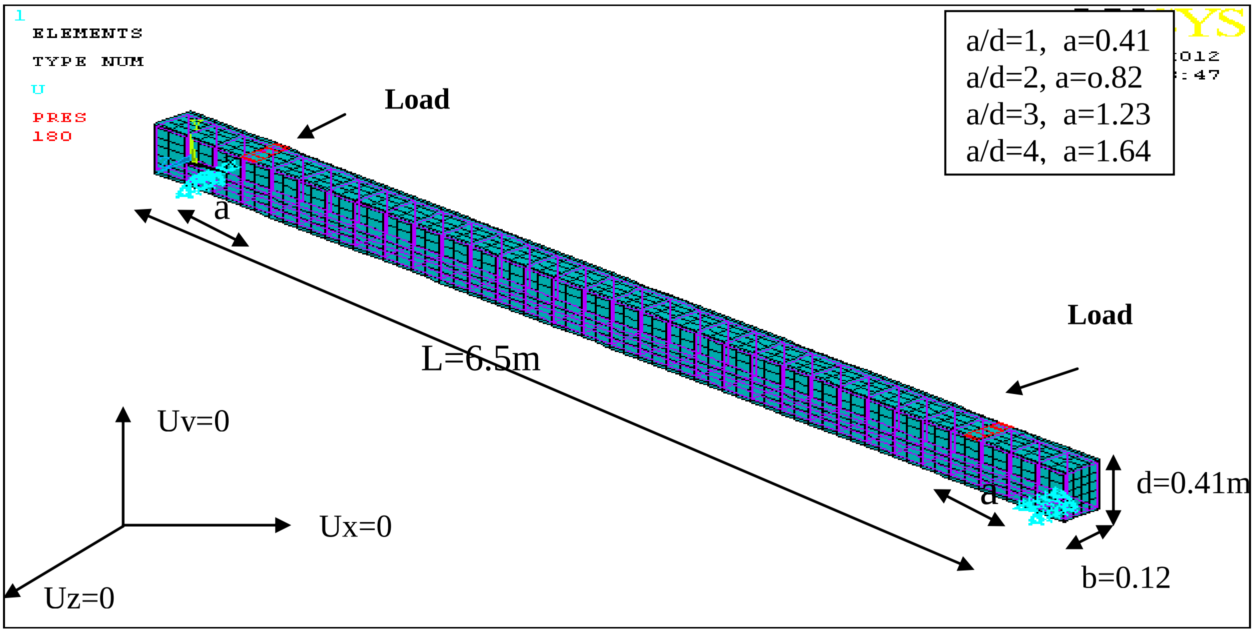


Figure (6) Finite element idealization of beams of group two

b- *Crack pattern*

Figures (7) to (10) show the crack pattern of the beam A_{21}, A_{22}, A_{23}, $A_{24,}$ respectively. In beam A_{21} (a/d=1) Inclined cracks appeared followed in the same time by crushing of concrete in the compression zone between the applied load and support, the mode of failure of this beam was compression failure. In beam A_{22} (a/d=2) the flexural –shear crack propagated gradually toward the loading point, and crushing of concrete occurred somewhat above the crack. Several local cracks occurred at the level of the bars .The failure of the beam was shear –flexural failure. In beam A_{23} (a/d=3) and beam A_{24} (a/d=4) with the increase of the load, vertical cracks appeared in the bottom of the middle zone and the distance between the support and the load, the mode of failure was flexural failure. A pre-mature compression failure and shear compression failure were accrued for beam with a=d and a=2d respectively before the beam reached to its ultimate moment capacity.

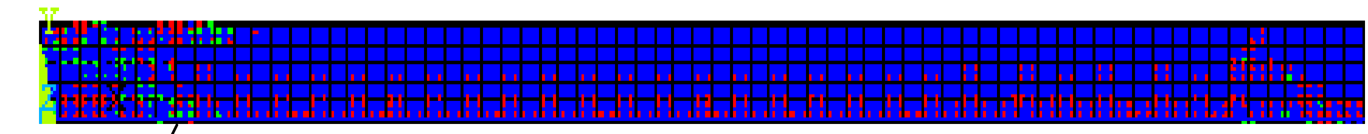


Figure (7) Crack patterns for beam A_{21} (a/d=1)

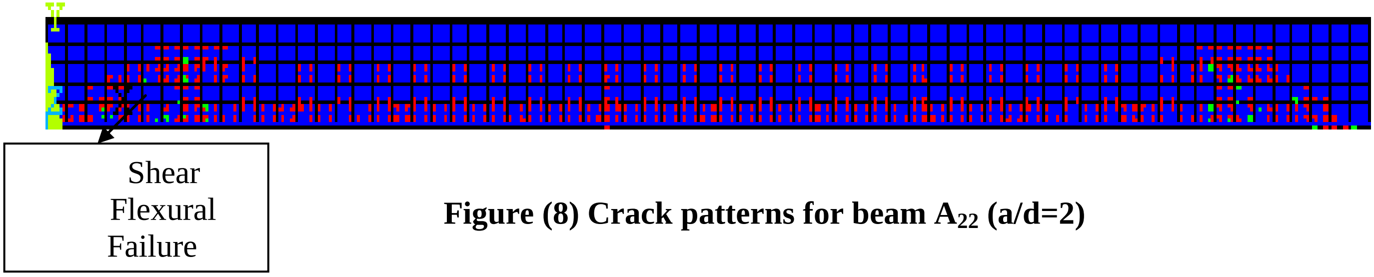


Figure (8) Crack patterns for beam A_{22} (a/d=2)

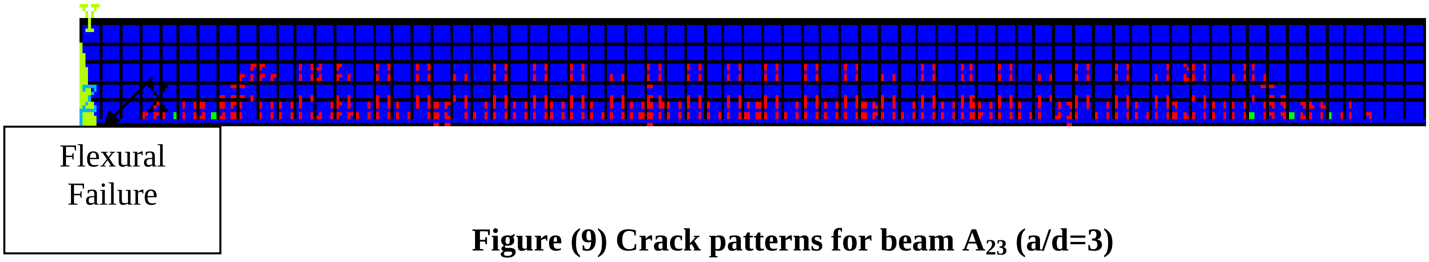


Figure (9) Crack patterns for beam A_{23} (a/d=3)

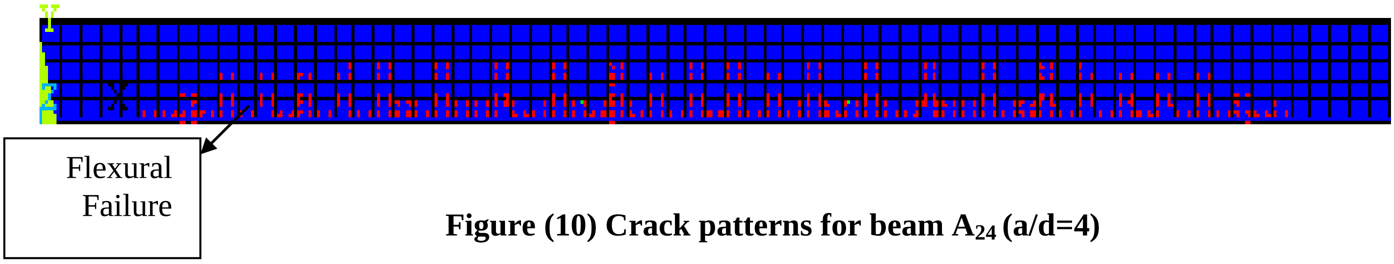


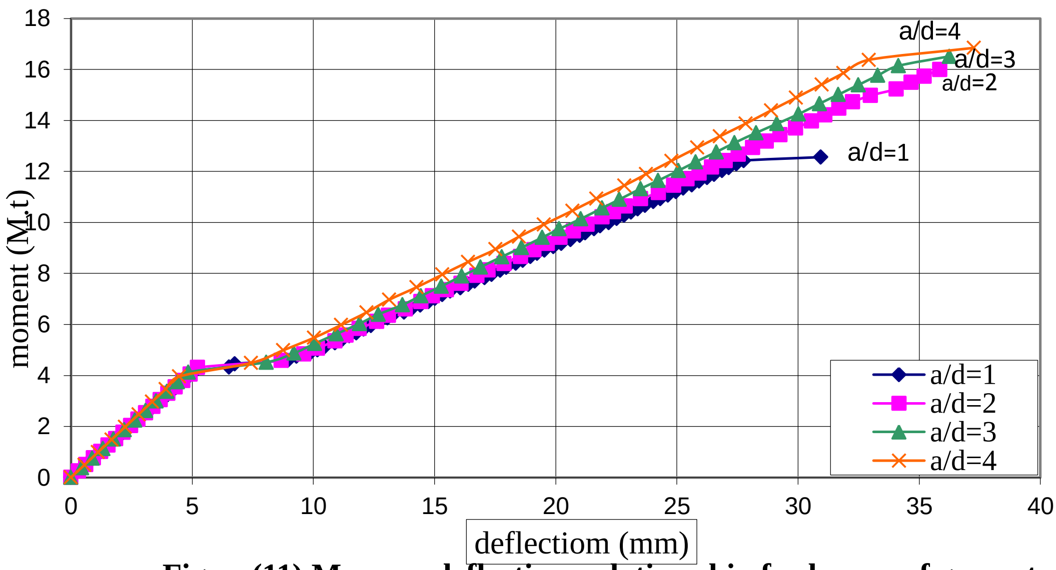
Figure (10) Crack patterns for beam A_{24} (a/d=4)

c- The moment –deflection relationship

Table (3) summarizes the theoretical results of the analyzed beams of group two .The variation of deflection with moment of all analyzed beams with different values of shear span –to-depth = (1, 2, 3 and 4) are shown in figure (11). The ultimate moment capacity was affected by the shear span –to-depth ratio, as the value of shear span –to-depth ratio increases the ultimate moment capacity increases as shown in Figure (12).

d- Code limitation for deflection

comparison was made between the predicated deflection at 0.67 of ultimate moment capacity which is considered as service moment using finite element method and the value recommended by code based on (span /250) As given in table(3). It can be seen from this comparison that the predicted value of deflection at o.67 of ultimate moment capacity less than recommended value based on the (span /250).



Figure(11) Momen -deflection relation ship for beams of group two

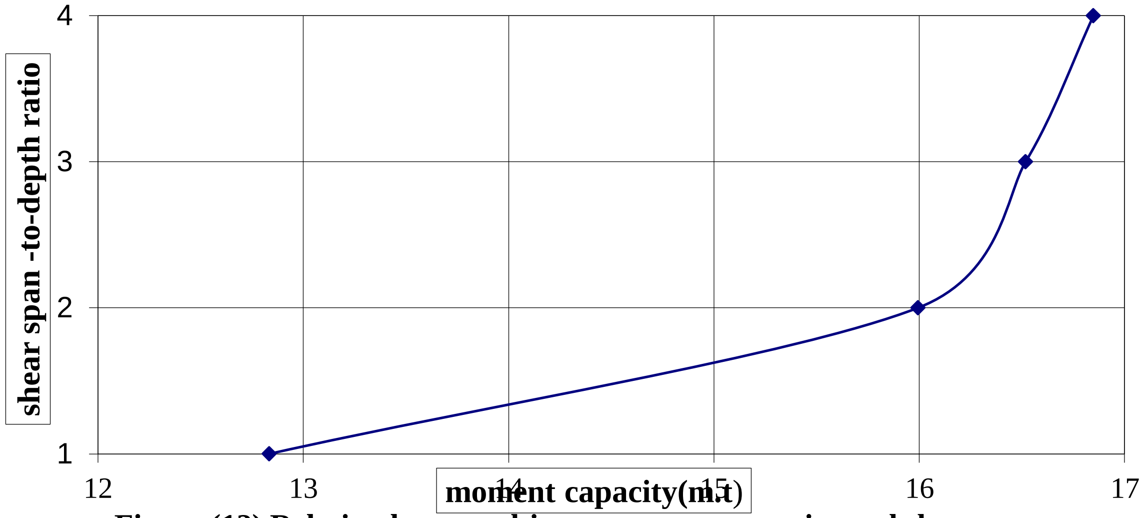


Figure (12) Relation btween ultimate moment capacity and shear span-to-depth ratio

Table (3) summarizes the theoretical results of the analyzed beams of second group

beam No	a (m)	$\frac{a}{d}$	M_{cr} (m.t)	M_u (m.t)	$\frac{Mcr}{Mu}$	Shape of failure	δ_{cr} (mm)	δ_s (mm)	δ_{code} (mm)	$\frac{\delta_s}{\delta_{code}}$
A_{21}	0.5	1	4.3	12.56	0.342	Compression failure at position	5.23	20.7	26	0.79
A_{22}	0.9	2		15.99	0.268	Shear flexural failure	5.23	24.05	26	0.93
A_{23}	1.3	3		16.51	0.26	flexural failure	5.23	24.25	26	0.93
A_{24}	1.7	4		16.84	0.255	flexural Failure	5.23	24.99	26	0.96

L=Length, a=shear length, d=depth, M_{cr}=cracking moment, Mu=ultimate moment, δ_{cr}= deflection at cracking moment, δ_s= deflection at 0.67 of ultimate moment, δ_{code}= deflection recommend by code (L\250).

5-3- The third group (tension steel):-

a- Details of the analyzed beams of group three

This group consisted of four beams, they were identical in cross section, the length and the shear length –to- depth ratio, with percentage of tension steel μ (1, 2.5, 4, and 6) for beam A_{31}, A_{32}, A_{33} and A_{34}, respectively .Figure (13) presents the geometry and the load of the beams, in addition to the finite element idealization.

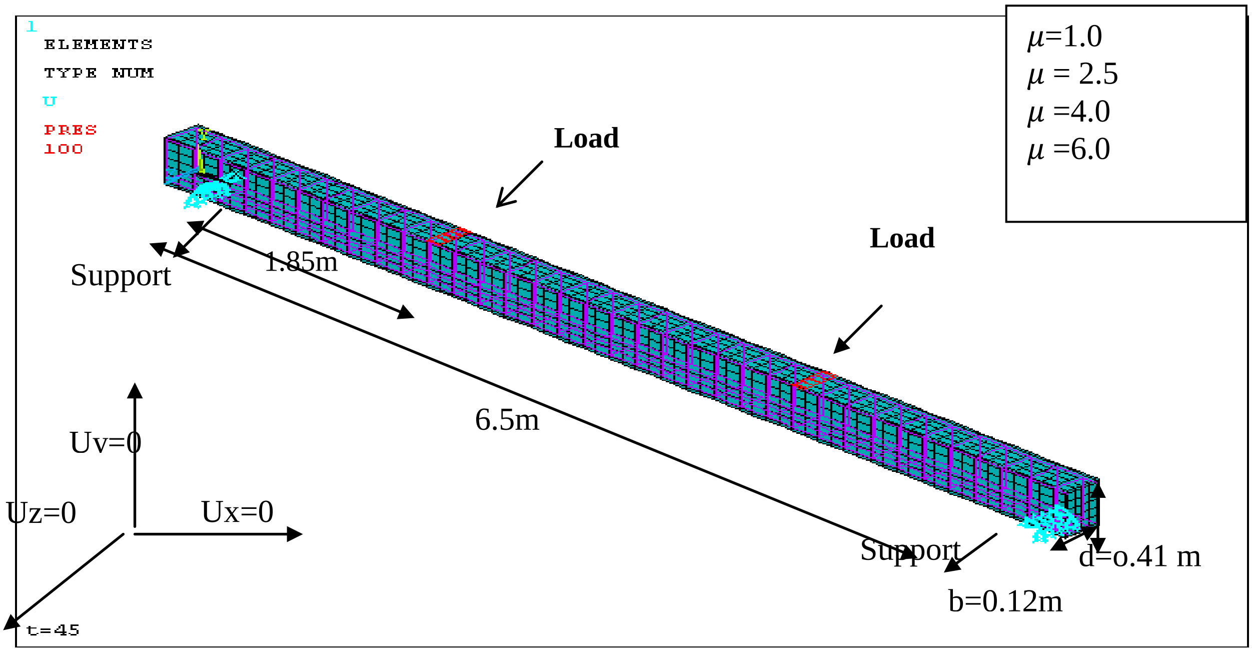


Figure (13) Finite element idealization of beams of group three

b- Crack pattern

Table (4) summarizes the theoretical results of the analyzed beams of the third group .Figure (14) presents the crack pattern of the tested beams of this group. since all beams had the same general shape of crack pattern as all cracks appeared at the bottom surface in the mid part of the beam and at the part between the load and the support. It can be seen from this figure that the mode of failure of this group was flexural failure.

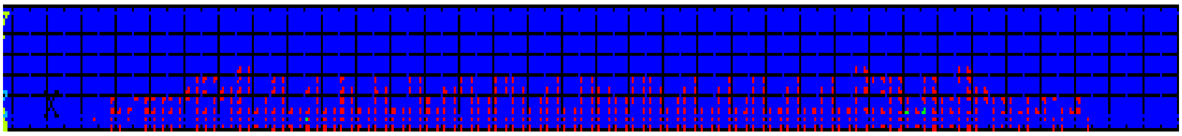


Figure (14) Crack patterns for beams of group three

C-Moment-deflection relationship

The variation of deflection with moment capacity of all analyzed beams with different values of percentage of steel = (1, 2.5, 4 and 6) are shown in figure (15). It can be seen that the deflection decreased as the percentage of steel in tension increased. Also it can be seen from figure (16) that the ultimate moment capacity was affected by the percentage of steel in tension as the value increases with the increasing of percentage of tension steel.

d - Code limitation for deflection

Comparison was made between the predicated deflection at 0.67 of ultimate load which is considered as service load using finite element method and the value recommended by code based on(span /250) As given in table(4). It can be seen from this comparisons that the predicted value of deflection at o.67 of ultimate moment capacity less than recommended value based on the (span /250).

Table (4) summarizes the theoretical results of the analyzed beams of third group.

beam No	μ	$\frac{a}{d}$	(M_{cr}) (m.t)	(M_u) (m.t)	$\frac{Mcr}{Mu}$	Shape of failure	δ_{cr} (mm)	δ_s (mm)	δ_{code} (mm)	$\frac{\delta_s}{\delta_{code}}$
A_{31}	1	4.5	3.96	9.35	0.42	flexural failure	4.17	22.98	26	0.88
A_{32}	2.5		4.44	16.87	0.26		4.67	21.9	26	0.84
A_{33}	4		5.25	20.52	0.255		4.77	19.96	26	0.76
A_{34}	6		6.84	27.36	0.25		5.13	17.55	26	0.675

L=Length, μ =percentage of steel in tension a=shear length, d=depth, M_{cr}=cracking moment, M_u=ultimate moment, δ_{cr}= deflection at cracking moment, δ_s= deflection at 0.67 of ultimate moment, δ_{code}= deflection recommend by code (L\250)

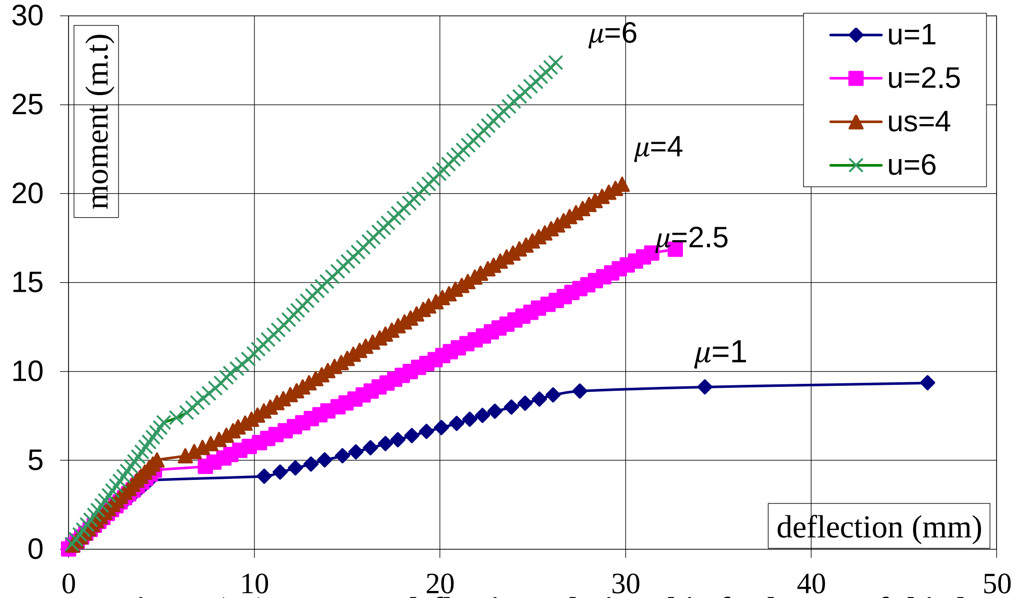
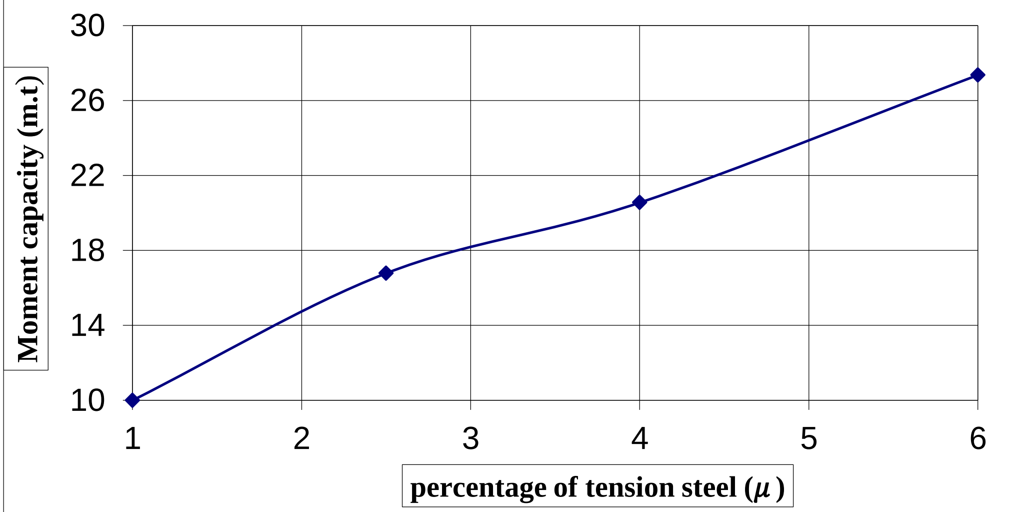


Figure (15) Moment-deflection relation ship for beams of third group



Figure(16) Relation between ultimate moment capacity and precentage of tension steel

6 - Conclusions:-

1- The service deflection in HSC beams of a rectangular section depends on the span –to- depth ratio, the shear span to depth ratio and the percentage of tension steel.

2- Increasing of the shear span to depth ratio has a major significant effect on the shape of failure of HSC.

3- The results showed that, as the ratio of theoretical cracking moment to theoretical ultimate near to 0.25, as long as the span to depth ratio less than 16, the beam become more ductile and the value of deflection within the code limitation.

4- The increasing of percentage of main steel more than the balanced steel leads to decreasing of deflection for the same load level and the behaviour become more brittle.

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دراسة نظرية لسلوك الكمرات اخرسانية عالية المقاومة تحت تأثير عزم الانحناء

ملخص البحث

يقدم هذا البحث دراسة نظرية لسلوك الكمرات الخرسانية عالية المقاومة ذات مقاومة (٧٥٠) كجم /سم٢ و القطاع المستطيل تحت تأثير عزم الانحناء باستخدام البرنامج غير الخطى للعناصر المحددة (Ansys) ولقد أجريت الدراسة على ثلاثة عشر نموذجا أخذا فى الاعتبار أهم العوامل التي تؤثر على سلوك الانحناء، مثل نسبة طول الكمرة إلى عمقها ونسبة طول القص إلى العمق و نسبة حديد التسليح في الشد . أوضحت الدراسة أن زيادة نسبة طول الكمرة الى عمقها له تأثير ايجابى على الممطولية وتأثير سلبى طفيف على عزم الانهيار، ومن ناحية أخرى وجد أن شكل وعزم الانهيار يتأثران بزيادة نسبة طول القص ألى العمق. فى حين أن زيادة نسبة حديد التسليح فى الشد له تأثير سلبى على الممطولية وتأثير ايجابى على عزم الانهيار وعزم التشريخ .